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APPENDIX G
DRL AF TM NO. 58

THE DISPENSING AND BEHAVIOR OF CHAFF IN SPACE

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THE UNIVERSITY OF TEXAS

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WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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I. INTRODUCTION

Chaff has been used extensively during the past two decades for establishing airborne radio scattering targets. The principal advantage of this material is the very high echo area per unit weight. Heretofore chaff employment has been confined largely to the lower portion of the earth's atmosphere. With the advent of man-made satellites and other types of space vehicles, it becomes worthwhile to consider some of the problems associated with the use of chaff above the earth's atmosphere.

II. SCOPE

This memorandum deals primarily with investigations conducted at DRL in two areas of the space-chaff problem. First, an experimental study was made to determine some possible methods of dispensing chaff at very high altitudes. Secondly, the behavior of chaff when dispensed from an earth satellite in a circular orbit was investigated.

Many uses for chaff in space have been suggested by groups throughout the country. Some possible applications include scatter communications, countermeasures, and decoys. It is not the intention of this paper, however, to deal with the uses of chaff in space or the quantities necessary for the various applications.

III. REQUIREMENTS FOR A VERY-HIGH-ALTITUDE DISPENSER

The basic requirements for a very-high-altitude dispenser are as follows:

(1) The dispenser must separate the chaff into individual dipoles without mutilation.

(2) The dispenser must give the dipoles pre-determined velocity with considerable precision.

For chaff to be most effective as an electro-magnetic reflector, it is necessary that the cloud be composed of individual dipoles, not groups or clusters. When chaff is dispensed from a fast-moving vehicle within the earth's atmosphere, drag forces tend to separate the chaff clusters into individual dipoles. Outside of the earth's atmosphere, the drag forces available for dipole separation are very small. Hence it is necessary to use some other means to assure efficient separation of the chaff bundle.

The velocity given to the dipoles when chaff is dispensed within the earth's atmosphere can be varied over a wide range and still produce a cloud of reasonable dimensions and dipole density. If the velocity is small, wind currents will disperse the chaff. If the initial velocity is large, the excess kinetic energy will be rapidly absorbed by the air. In any event, the cloud will retain finite dimensions for some appreciable length of time. On the other hand, chaff dispensed above the earth's atmosphere (but not in orbit) will retain the dispensing velocity indefinitely. If this velocity is large the cloud will grow to tremendous proportions and become quite diffuse in a short time. If the dispensing

velocity is small, the bloom time* will be excessive. For chaff dispensed from a vehicle in orbit, the magnitude and direction of the dispensing velocity will determine the rate of growth of the earth chaff belt as well as the maximum dimension (width, depth, etc.) of the belt. More will be said about the behavior of orbiting chaff in later sections of this report.

*The time required for the cloud to reach useful size.

IV. EXPERIMENTAL INVESTIGATION OF DISPENSING METHODS

A. Vapor-Pressure Method

1. General description:

This technique utilizes the rapid vaporization and expansion of fluids to achieve the initial dipole separation and small velocity differential desired. The actual mechanics of the operation consists of soaking the dipoles thoroughly with the desired fluid and packing them into a dispensing chamber. After all excess fluid is drained off, the package is then immediately sealed under atmospheric conditions. When it is to be dispensed, the fluid-soaked chaff is thrust from the dispensing chamber into a vacuum. This vacuum causes rapid vaporization and consequent expansion of the fluid. The vapor flow imparts velocity to the dipoles and the presence of fluid between individual dipoles ensures good dipole separation.

2. Scope of experimental work:

The experimental work was limited to observation of dipole separation and measurement of cloud velocity as a function of fluid vapor pressure and chaff type. The fluids used ranged from Dow Corning 200 silicone fluid, 1.0 centistoke viscosity with a vapor pressure at ambient temperature of 2.7 mm Hg, to acetone, which has an ambient temperature vapor pressure of 229 mm Hg.

Four types of aluminum and one of glass chaff were used in the experimental work. Aluminum chaff size ranged from 0.75 x 0.016 x 0.0005 in. to 4.5 x 0.25 x 0.0005 in. The length and diameter of the glass dipoles were 0.75 in. and 0.002 in. respectively.

3. Description of test equipment:

The bell jar (volume approximately 3 ft^3) and one of the ejection devices used in these investigations are shown in Figure 1. Figure 2 is a cross section drawing of the ejection apparatus and shows the method of executing chaff ejection. The chaff is exposed to the vacuum by a downward displacement of the enclosing cup. This movement is accomplished by pulling on the center rod shown in Figure 1.

It should be noted that two different types of chaff containers were employed. These containers are shown in Figure 3. The upper container was used with the 3/4" length dipoles which were packed axially. The lower container was used with the 4-1/2" length dipoles which were packed circumferentially. Both containers were designed for radial chaff ejection.

A pictorial recording of each test was made with a Fastax camera.

4. Test procedure:

The chaff to be used in the test was bathed and cleaned thoroughly in the fluid involved. This was done to remove the chaff lacquer coating which is soluble in most of the fluids under consideration. (This procedure was not necessary for the 4-1/2" chaff which has no lacquer coating.)

After the bathing operation the chaff was packed into the appropriate chaff container. It was then soaked thoroughly with fluid, the excess fluid poured off, and the container sealed immediately. The loaded container was then placed in the bell jar.

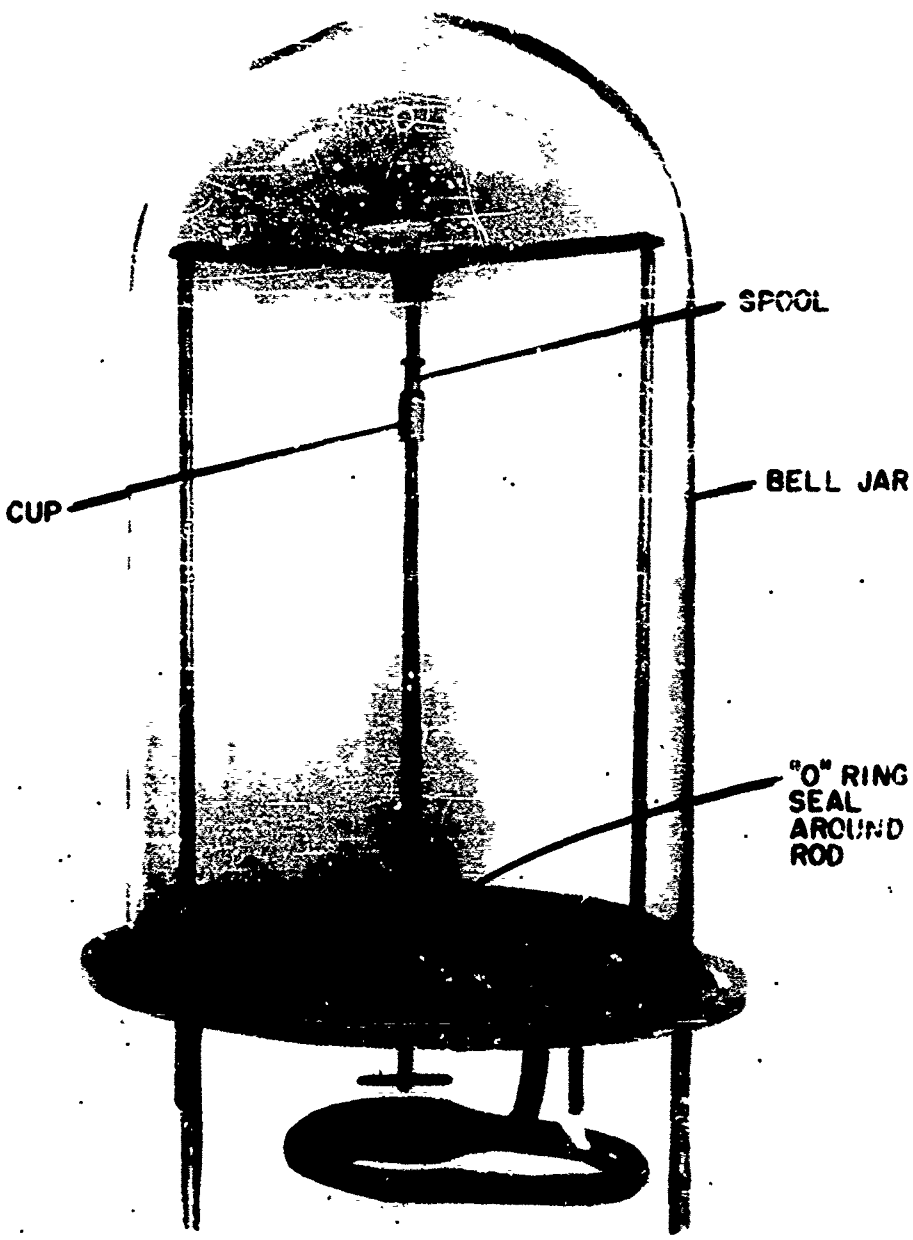


FIGURE 1
TEST SET-UP BEING USED IN VERY-HIGH-ALTITUDE CHAFF-DISPENSING STUDIES

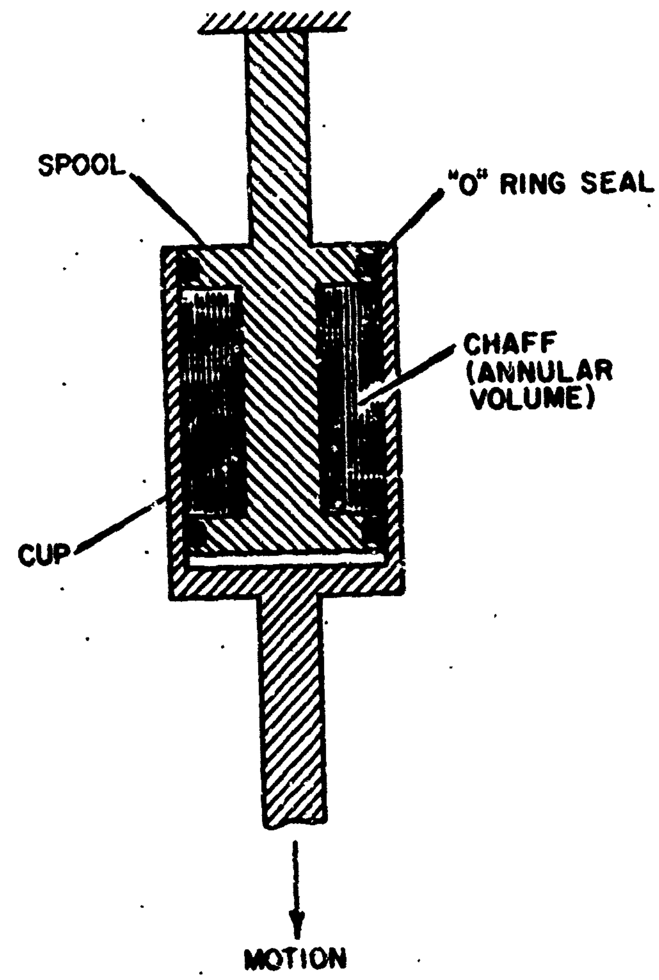
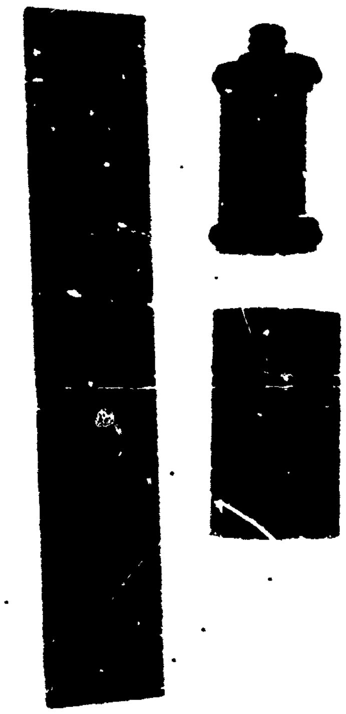


FIGURE 2
CHAFF CONTAINER FOR SIMULATED HIGH ALTITUDE DISPENSING EXPERIMENTS



(a) CONTAINER FOR AXIALLY PACKED CHAFF.

(b) CONTAINER FOR CIRCUMFERENTIALLY PACKED CHAFF.

FIGURE 3

Care was taken to prevent radiant heating of the fluid and chaff from the high intensity Fastax lights during the tests. All the tests were run at bell-jar pressures of less than 0.10 mm Hg. Precautions were taken to prevent pressure leakage around the O-ring seals of the chaff container.

5. Data reduction:

The Fastax film of the tests was first studied qualitatively with a 16 mm movie projector to observe dipole dispersion and separation. It was then studied quantitatively frame-by-frame to determine dipole velocity. Basically, the velocity was determined by measuring the change in diameter of the cloud for a given number of frames. Knowing frame speed and an appropriate dimensional scale factor, the average dipole velocity could be readily calculated.

6. Results:

The principal results of the experimental investigations are given in Figure 4. Note that the dipole velocity is approximately linear with $\sqrt{\frac{pA}{m}}$. In plotting these data, several very irregular points have been omitted.

A complete description of all tests is given in tabular form in Appendix A.

The results of Figure 4 stated in equation form are:

$$V = 2.03 + 0.0394\sqrt{\frac{pA}{m}}$$

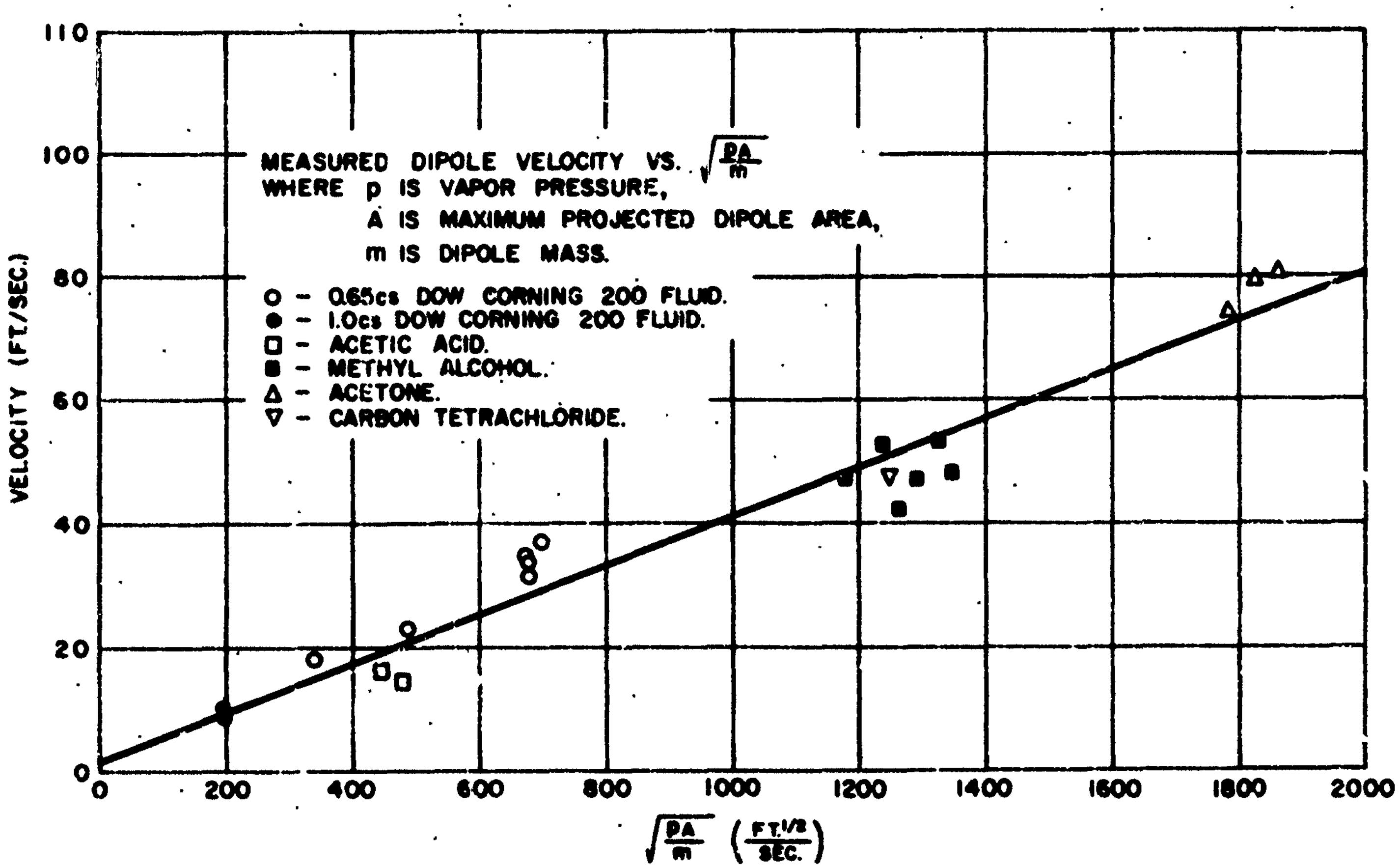


FIGURE 4

EXPERIMENTAL RESULTS

The coefficients were determined from a least squares fit of the experimental points in Figure 4. It should be stressed that this is strictly an empirical statement. It is approximate, and is known to hold only for the vapor-pressure range of the table in Appendix A. It should be used with caution outside this range.

Figure 5 is a print of several frames of film from Test K which employed methyl alcohol as the ejecting fluid. Figure 6 (2 pages) is a similar print of Test W which employed no fluid. Note the great difference in the behavior of the two chaff clouds.

The frame speed for both runs was approximately 2300 frames/sec, but only every tenth frame was printed. The horizontal bar appearing in Figures 5 and 6 was used for calibration in the determination of dipole velocity.

7. Limitations:

The use of fluid vaporization to dispense chaff is limited to very low pressure or high-altitude conditions. It is necessary that the ambient pressure be very low compared to the vapor pressure of the fluid in use.

Since vapor pressure varies radically with fluid temperature, knowledge of temperature conditions at the time of dispensing is necessary. The unevaporated fluid is cooled during dispensing as vaporization takes place. However, the conditions in the test chamber and in space are essentially the same in this respect. The radiation loss to space during the very short dispensing period is negligible.

Dipole separation may be incomplete with a fluid of relatively low vapor pressures. In the tests using 1.0 centistoke viscosity Dow

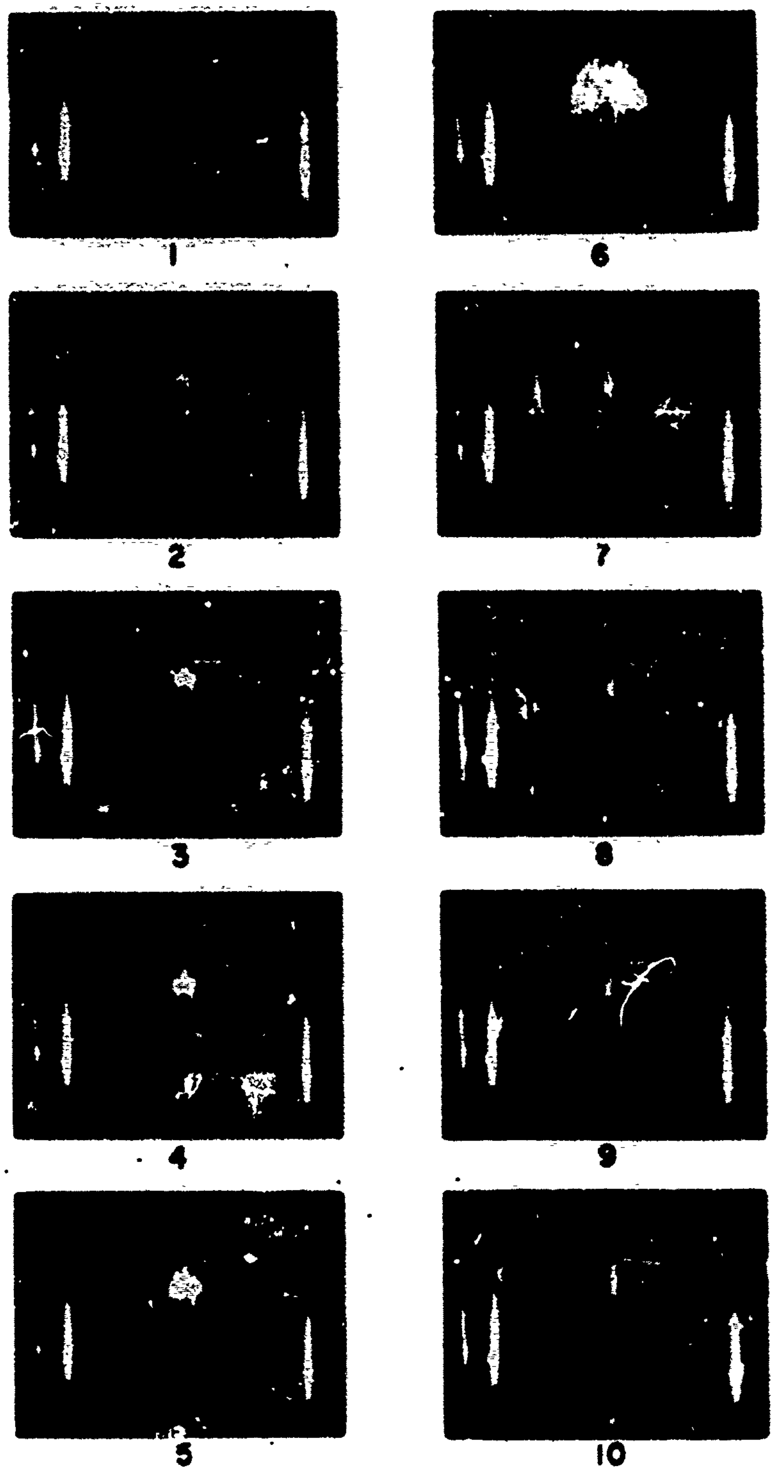


FIGURE 5
CHAFF EJECTION USING METHYL ALCOHOL AS EJECTING FLUID

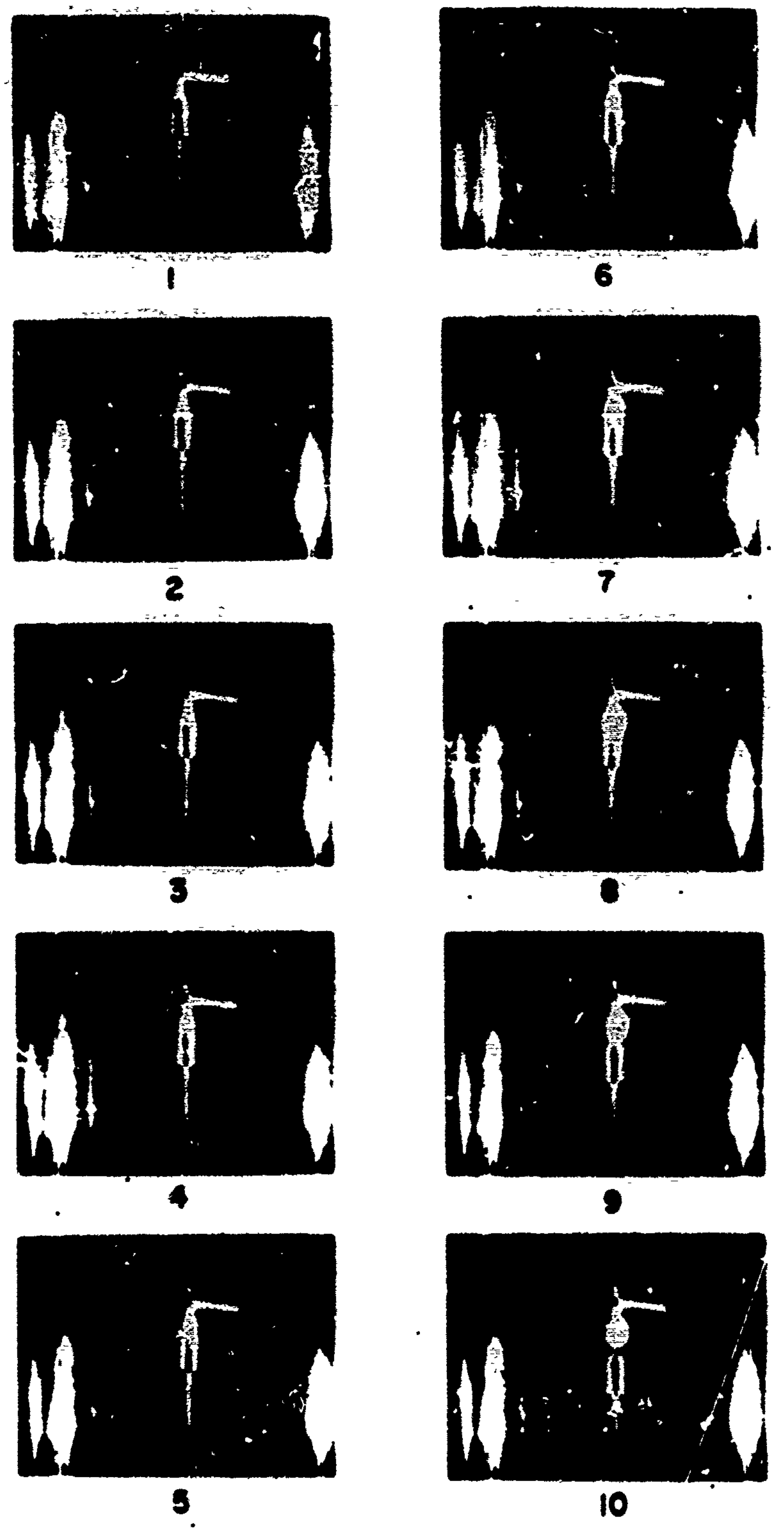


FIGURE 6.
CHAFF EJECTION USING NO EJECTING FLUID

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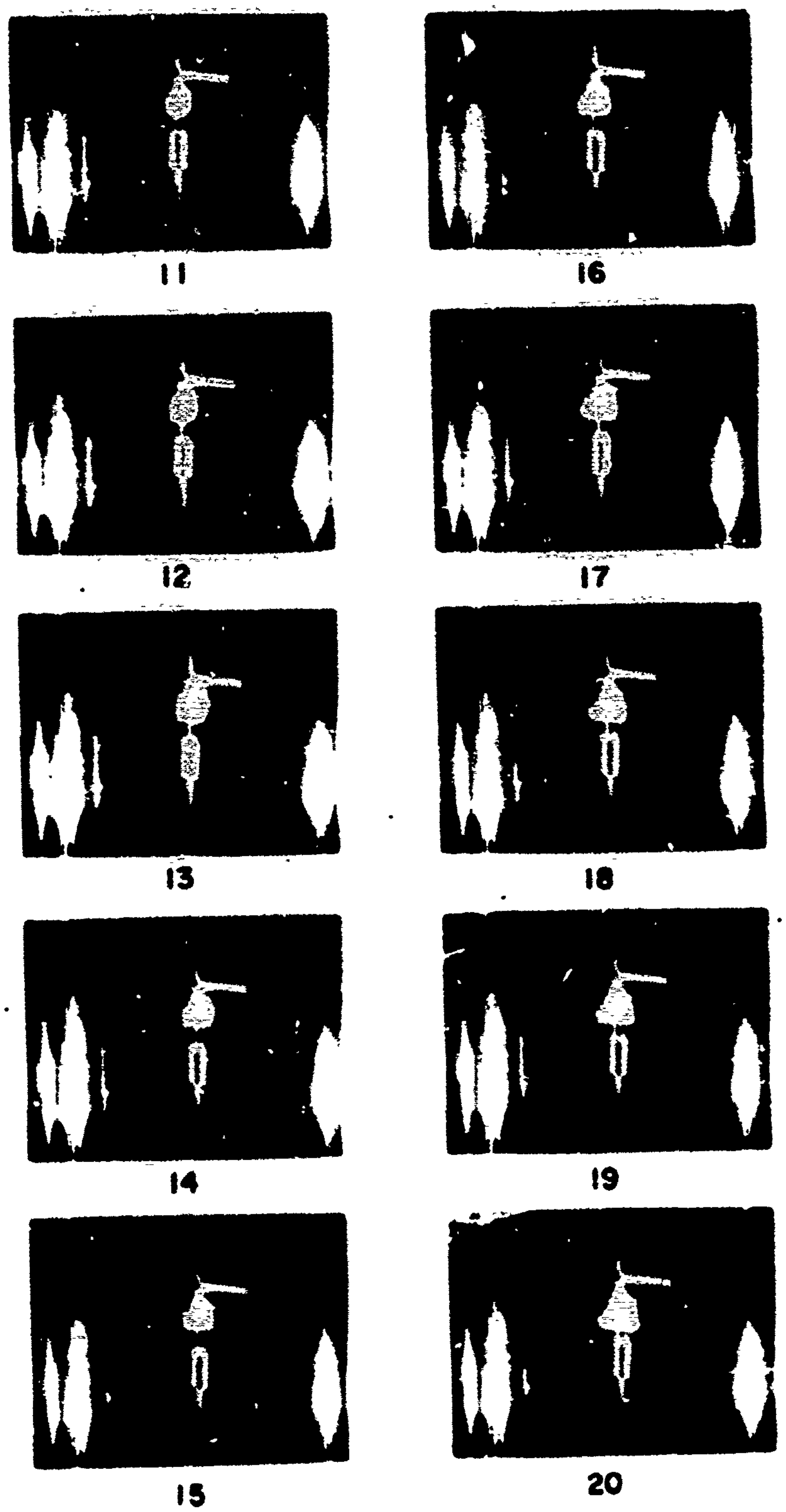


FIGURE 6 (CONTINUED)

Corning 200 Silicone fluid (vapor pressure about 2.7 mm Hg at 72°F), some dipoles stuck together in clumps of three or four.

This method of dispensing is not applicable to low-frequency "rope" type chaff. Other specialized methods of dispensing will be necessary for this variety of chaff.

B. Low-Pressure Gas Dispenser

Another method of chaff dispensing investigated involved a low-pressure gas dispenser. This dispenser consists of a glass cylinder packed with chaff and pressurized with low-pressure air. The chaff is dispensed by shattering the glass with bullets fired by high-pressure air as shown in Figure 7.

Figure 8 shows one of the dispensers designed and built at DRL. Several units were tested but sealing difficulties, particularly at the ends, were encountered. The project was discontinued in favor of concentration on the vapor-pressure method.

C. Spin Dispensers

The spin dispenser is a mechanical device for ejecting and dispensing the chaff, as shown in Figure 9. Three dispensers of this nature were designed and built at Defense Research Laboratory and installed as "piggyback" equipment in a Thor missile nose-cone. The missile was fired from Cape Canaveral in January of 1960. For a complete description of this experiment and the associated equipment see DRL AF Technical Memorandum No. 50 (Contract AF 33(616)-5164).

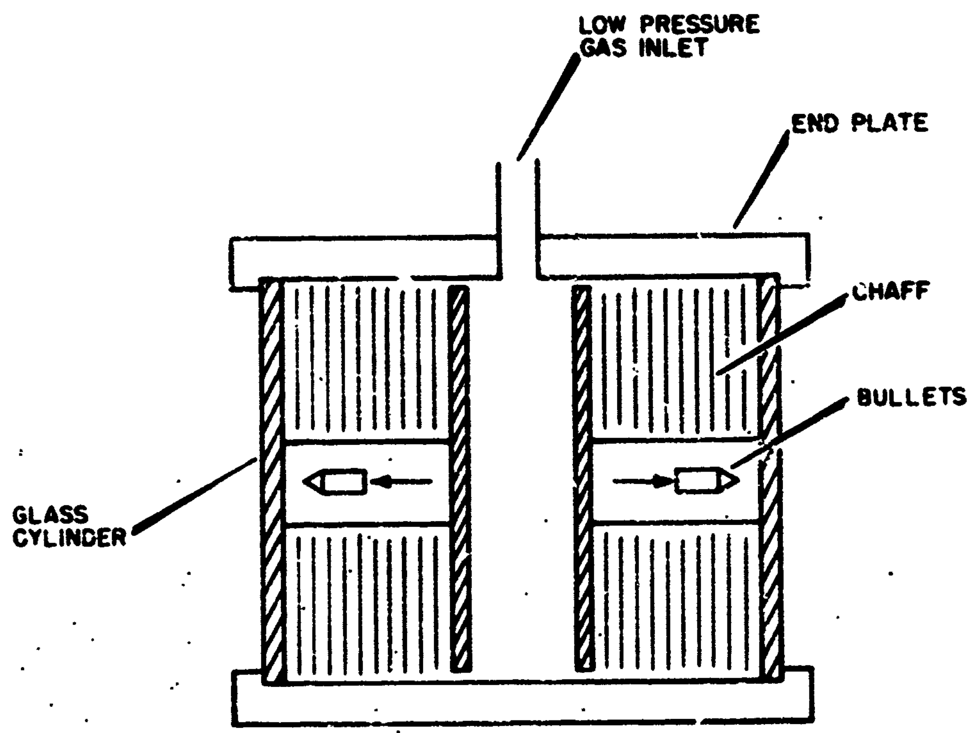


FIG. 7
LOW-PRESSURE GAS CHAFF EJECTION SETUP

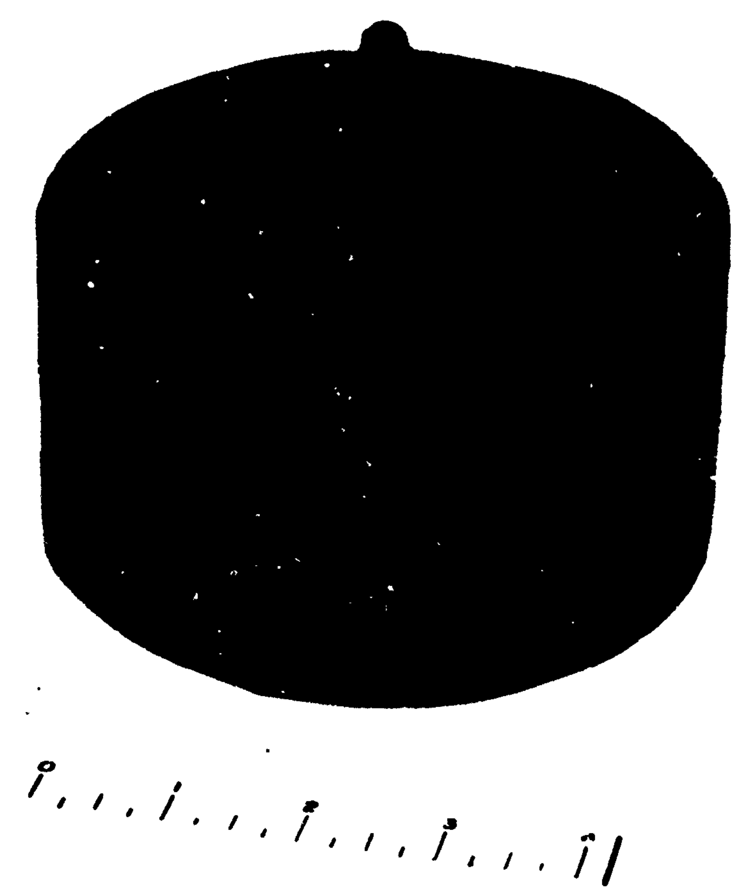


FIGURE 8
CHAFF CONTAINER FOR LOW PRESSURE GAS EJECTION

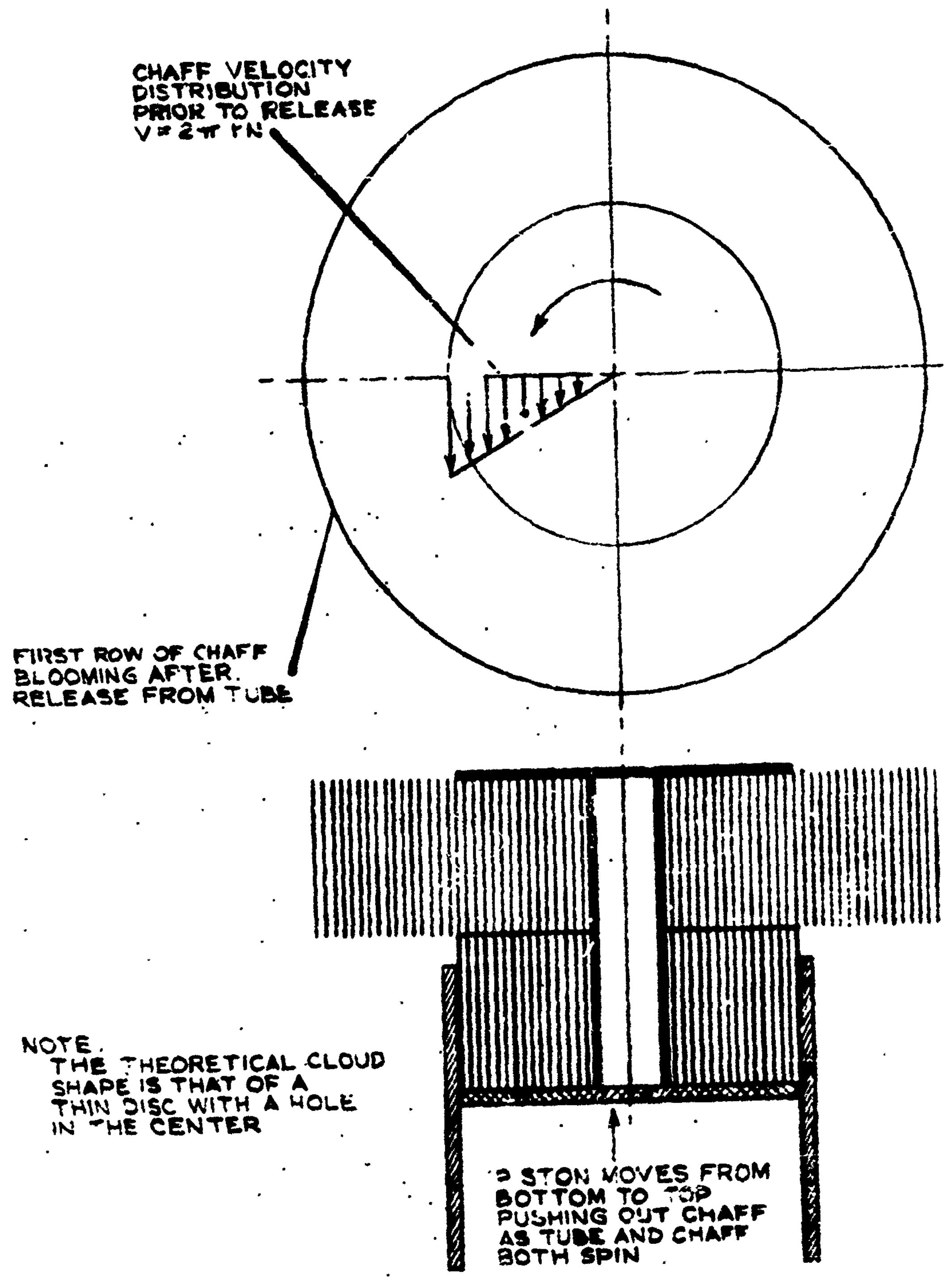


FIG. 9 - SPIN-DISPENSING TECHNIQUE

V. THEORETICAL INVESTIGATIONS OF CHAFF BEHAVIOR IN SPACE

A. Scope of Study

Theoretical studies have been limited to the behavior of chaff when dispensed from a vehicle in a circular geocentric orbit. From curves and other information included herein, it is easy to determine the effect of the dispensing velocity on certain pertinent chaff cloud parameters. Of particular interest is the time required to form a complete belt of chaff around the earth.

B. Assumptions

In the theoretical analysis the following assumptions were made:

(1) Drag on the dipoles is negligible during the period of interest. This will be true if the orbit is sufficiently high above the earth.

(2) The dispensing velocity is imparted to the dipoles instantaneously. Experimental results from the vapor-pressure study show that the velocity is obtained within a few milliseconds.

(3) The simplified two-body equations are sufficiently accurate to describe the orbit of the dipoles. This will be true so long as the orbit is near enough to the earth that the attraction to the moon, sun, and planets can be neglected. The effects of earth oblateness are also neglected.

(4) Photoelectric and magnetic effects are negligible.

C. The Elliptical Orbit

Figure 10 shows an ellipse and defines some of its important properties. The path of a particle in orbit about the earth is an ellipse with the earth at one focus. A circular orbit is a special ellipse for which the eccentricity is zero. (A discussion of orbits can be found in any good book on mechanics.)

Some of the more important relationships for simple two-body systems are given below.

(1)	$a = \frac{-\mu}{2E}$ where	$\mu = g_o R^2$ = constant
(2)	$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$	g_o = Acceleration of gravity at surface of earth-ft/sec^2
(3)	$E = 1/2\, V^2 - \frac{\mu}{r}$	R = Radius of earth - ft
	(Constant for a particular orbit)	E = Sum of kinetic and potential energy per unit mass (E<0 for ellipse)
(4)	$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$	h = Angular momentum per unit mass
(5)	$\tau = \frac{2\pi\mu}{\sqrt{(-2E)^3}}$	τ = periodic time - sec

D. Dispensing Effects

1. Preliminary Consideration

Consider a vehicle in a circular geocentric orbit capable of dispensing chaff with a velocity ΔV relative to itself in all directions simultaneously. (See Figure 11.)* After dispensing, each dipole will have

*Throughout this discussion, the subscript 0 wherever used refers to the conditions of the original circular orbit. The subscript 1 refers to the orbital conditions produced by ΔV_1 and so forth.

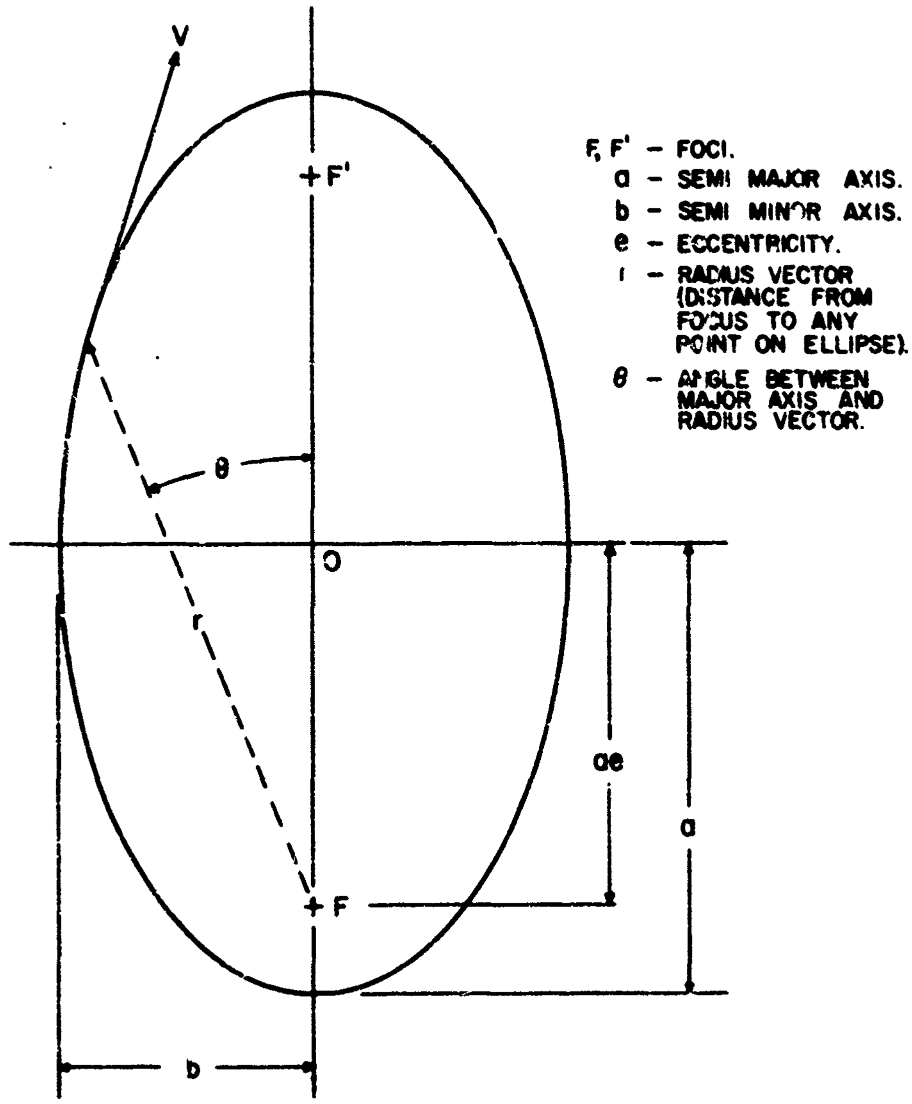
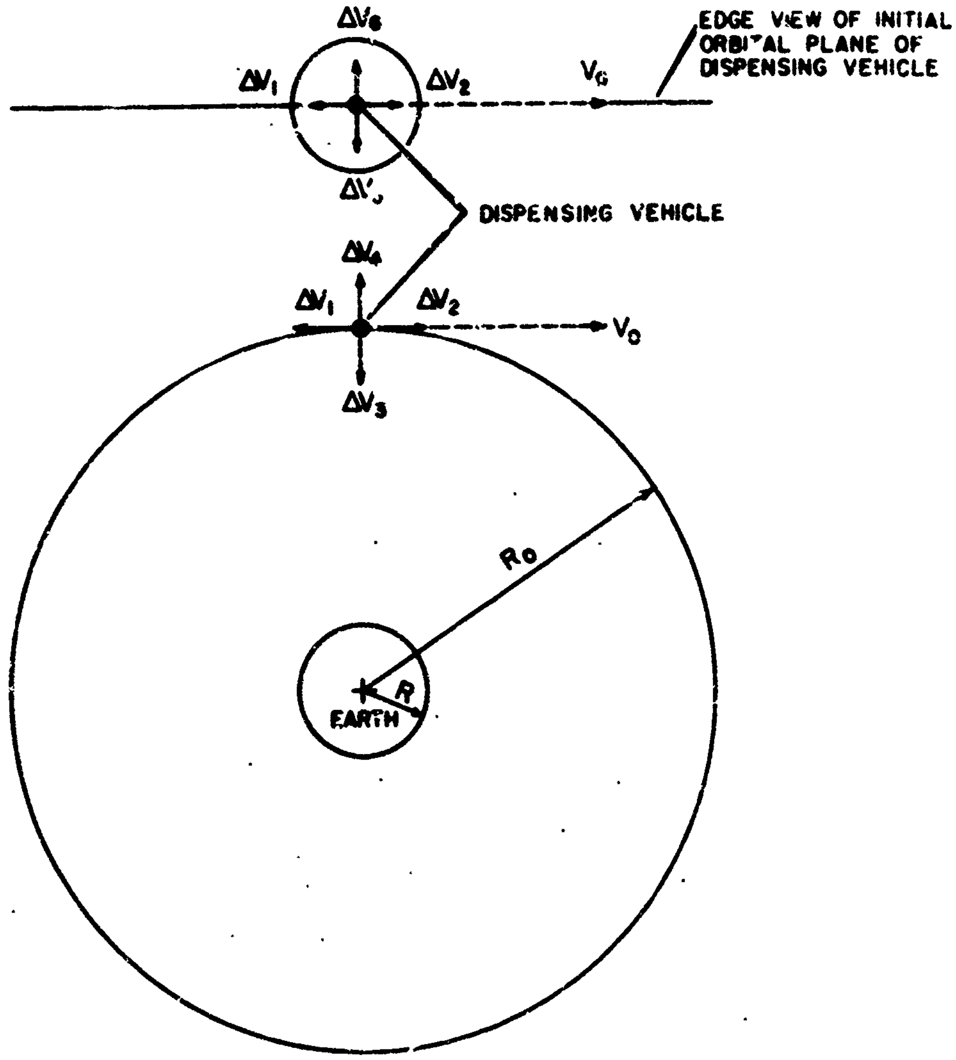


FIGURE 10
THE ELLIPTICAL ORBIT



V_0 = INITIAL CIRCULAR ORBITAL VELOCITY.
R_0 = INITIAL ORBITAL RADIUS.
R = RADIUS OF EARTH = 20.925×10^6 FT.
ΔV = PERTURBATION OR DISPENSING VELOCITY.

$$|\Delta V_1| = |\Delta V_2| = |\Delta V_3| = |\Delta V_4| = |\Delta V_5| = |\Delta V_6|$$

FIGURE 11

LIMITING DISPENSING CONDITIONS

its total velocity vector changed in magnitude or direction or both. Each dipole will theoretically be transferred to a unique elliptical orbit.

It is difficult to define as a function of time the shape and size of a group of orbiting dipoles when dispensed in the manner described in the above paragraph. In order to simplify the problem, only the dimensional limits of the chaff cloud were studied. The ΔV's of the directions shown in Figure 11 establish the maximum dimensions (width, depth, etc.) of the chaff cloud. It is now worthwhile to consider in detail the effect of these limiting ΔV's on the orbit parameters.

2. ΔV_1 and ΔV_2

The time required to "belt" the earth as well as the maximum depth of the belt are determined by ΔV_1 and ΔV_2 Figure 12 shows qualitatively the effect of ΔV_1 and ΔV_2 with respect to the initial circular orbit. A dipole ejected with velocity ΔV_1 will have a total velocity (immediately following ejection) and energy per unit mass less than that of the dispensing vehicle. On the other hand, ΔV_2 increases the dipole velocity and energy per unit mass. The vector addition of V_o to the various ΔV's shows clearly that ΔV_1 and ΔV_2 produce the extremes in dipole energy change. Since the periodic time is a function of E only, ΔV_1 and ΔV_2 also produce the extremes in periodic time (See Equation 5). It is these extremes in the period that determine the time required for the chaff cloud to "belt" the earth. It should be noted that the orbital plane of dipoles dispensed with ΔV_1 and ΔV_2 is the same as that of the dispensing vehicle.

Using Figures 13 and 14, it is possible to determine the number of orbits as well as the time required to form a complete earth belt

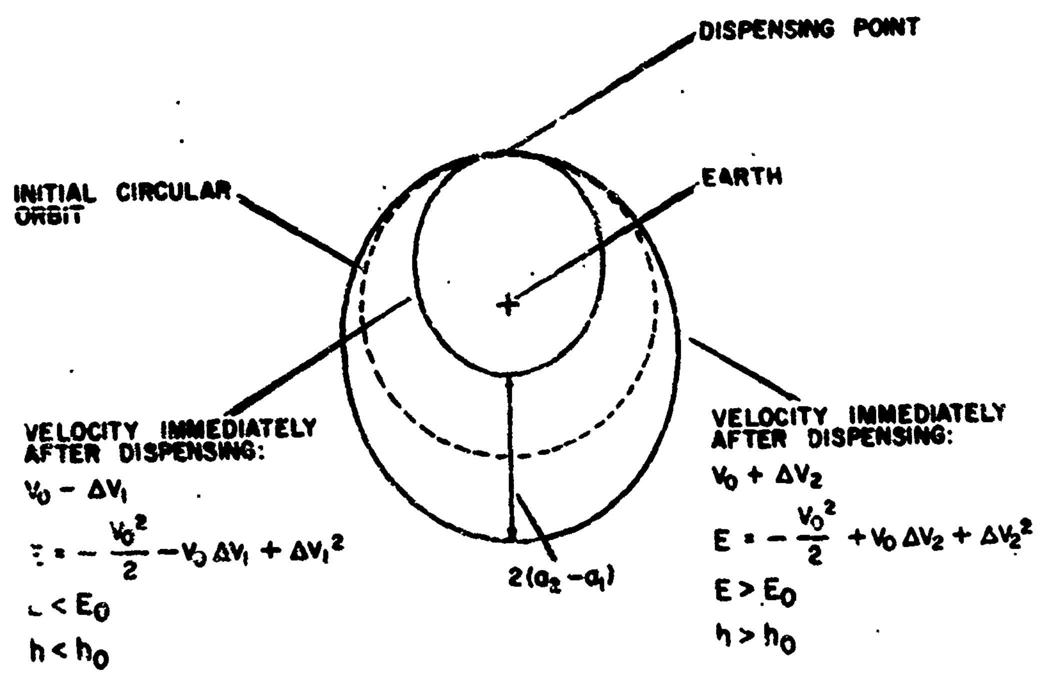


FIGURE 12
QUALITATIVE ORBITS AFTER PERTURBATION
FOR ΔV_1 AND ΔV_2

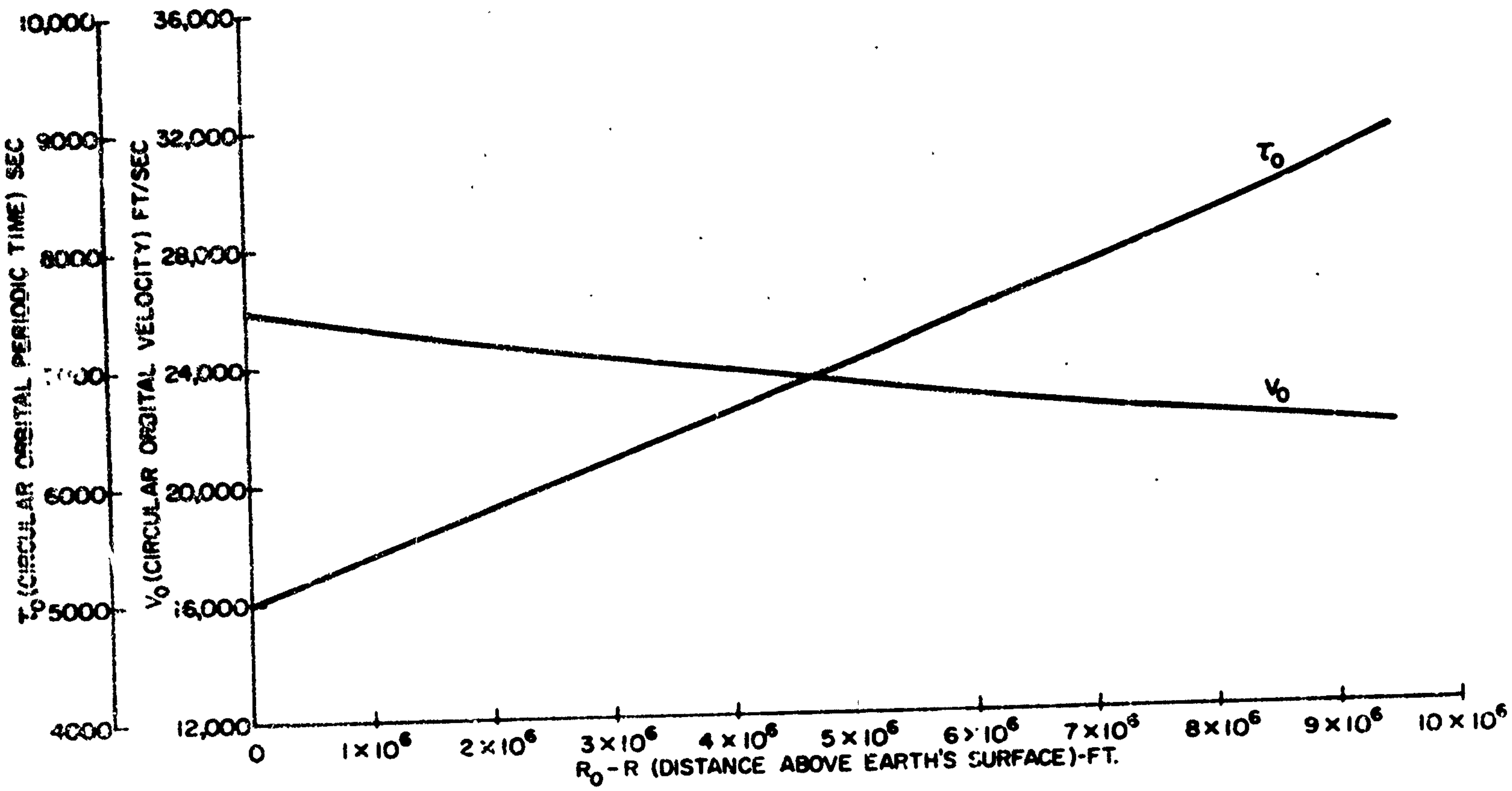


FIGURE 13
PERIOD AND VELOCITY OF A SATELLITE IN A CIRCULAR GEOCENTRIC ORBIT

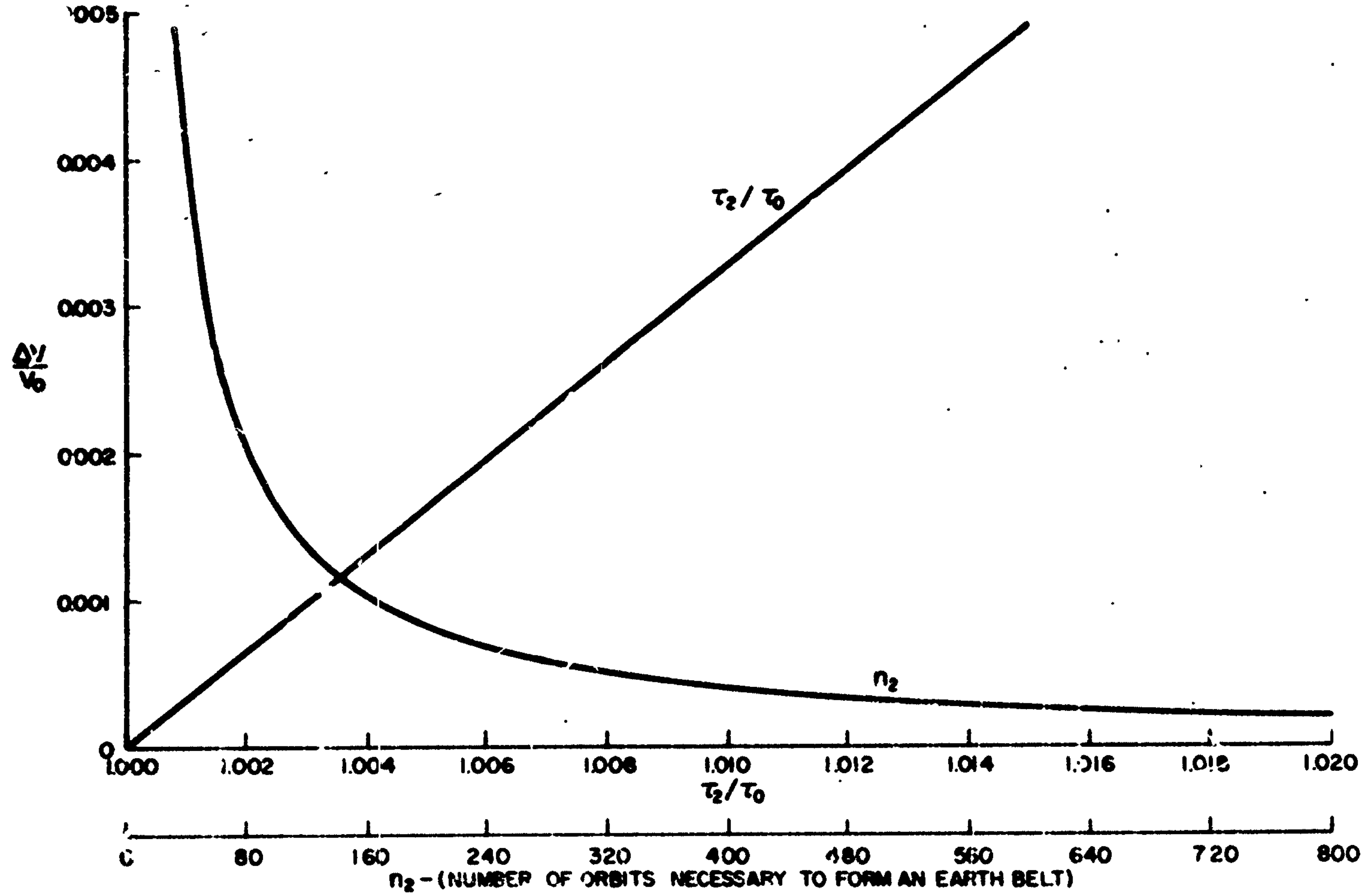


FIGURE 14
DIMENSIONLESS PARAMETERS PERTAINING TO CHAFF BELT FORMATION TIME

when the chaff is dispensed with ΔV limits shown in Figure 11. It is interesting to note that n (Figure 14) is a function only of $\frac{\Delta V}{V}$ and not R_o.*

The use of these curves can best be explained by an example.

Given

$\Delta V = 30$ ft/sec (determined by fluid vapor pressure)

$R_o - R = 6 \times 10^6$ feet

From Figure 13 for circular orbits

$V_o = 22{,}800$ ft/sec

$\tau_o = 7400$ sec

Then $\frac{\Delta V}{V_o} = \frac{30}{22{,}800} = 0.00132$

From Figure 14

$n_2 = 125$ orbits

$\frac{\tau_2}{\tau_o} = 1.004$

$\tau_2 = 1.004\ (7400) = 7440$ sec

Time required to belt the earth

$$= n_2\tau_2 = \frac{7440\ (125)}{3600} = 258 \text{ hours}$$

From Equation 1 it is readily seen that ΔV_1 and ΔV_2 also determine the maximum depth of the chaff belt. This thickness is given

*For a proof of this and for various derivations, see Appendix B.

by 2 (a_2 - a_1) as shown in Figure 12. This parameter is plotted in Figure 15 as a function of $\frac{\Delta V}{V_o}$.* Again the use of this curve can best be explained by an example.

Given (from above example)

$$\frac{\Delta V}{V_o} = 0.00132$$

$$R_o = 6 \times 10^6 + 20.925 \times 10^6 \text{ ft}$$

From Figure 15

$$\frac{2(a_2 - a_1)}{R_o} = 0.0105$$

$$2(a_2 - a_1) = 0.0105 \, (26.925 \times 10^6)$$

$$= 28.25 \times 10^4 \text{ ft}$$

$$= 53.5 \text{ miles}$$

3. ΔV_3 and ΔV_4

Figure 16 shows qualitatively the result of ΔV_3 and ΔV_4 with respect to the initial circular orbit. Note that the apse lines of these two orbits are shifted by plus and minus 90° from the dispensing point.** It is easily shown from energy considerations that these orbits do not produce the maximum cloud depth nor the extremes in orbital period. The orbital plane of dipoles dispensed with ΔV_3 and ΔV_4 is the same as that of the dispensing vehicle.

* See Appendix B for derivations.
**See Appendix B for proof.

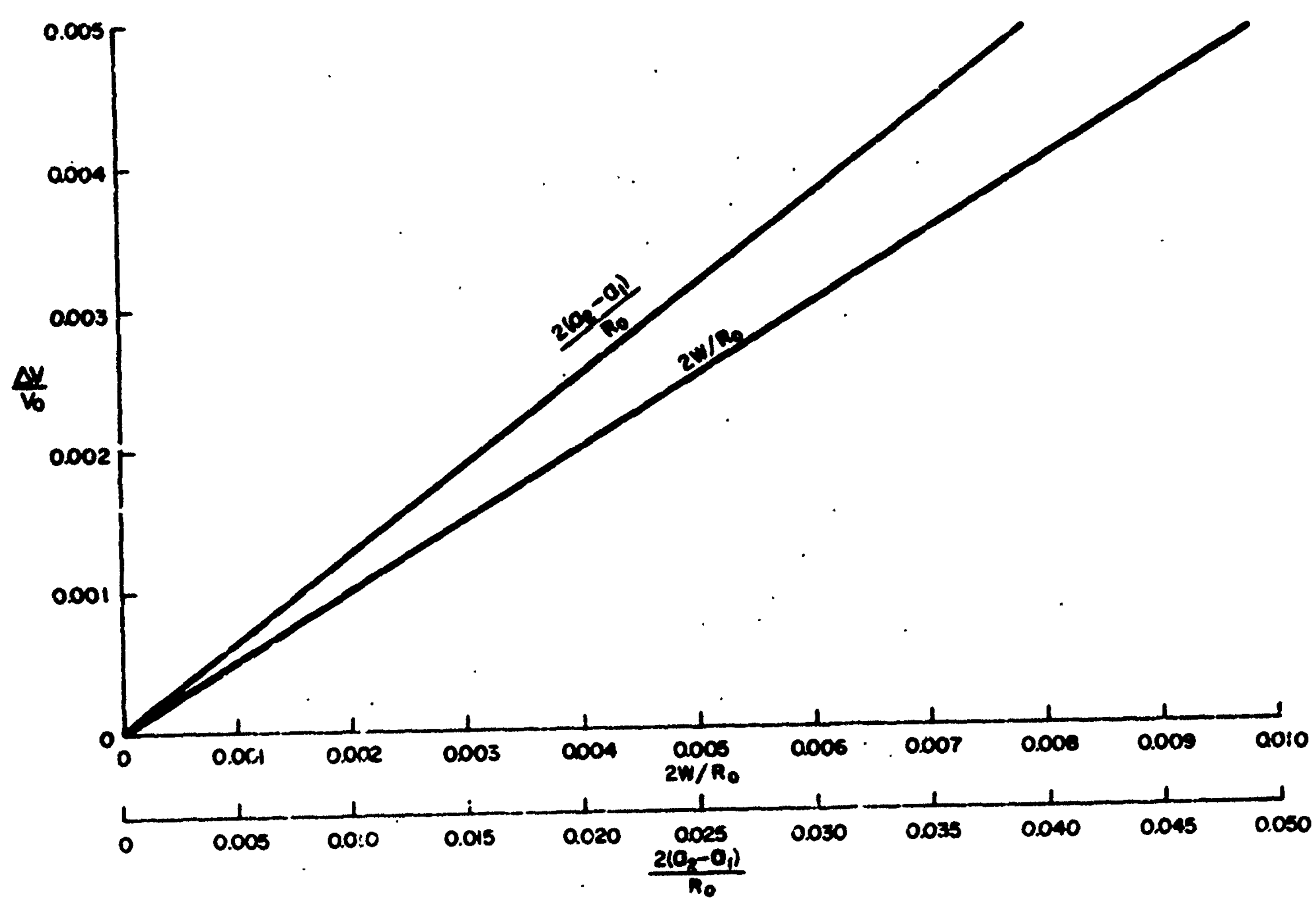


FIGURE 15
DIMENSIONLESS PARAMETERS PERTAINING TO CHAFF BELT DEPTH AND WIDTH

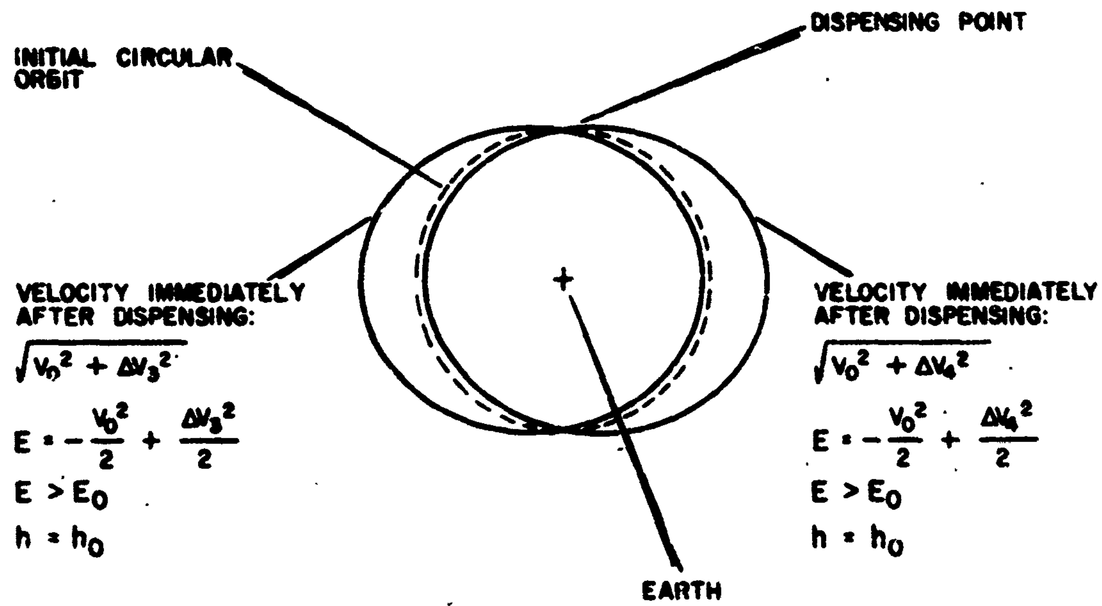


FIGURE 16

QUALITATIVE ORBITS AFTER PERTURBATION FOR ΔV_3 AND ΔV_4

4. ΔV_5 and ΔV_6

The maximum width of the chaff belt is determined by ΔV_5 and ΔV_6. Figure 17 shows qualitatively the effect of these dispensing velocities. The principal result is to rotate the planes of the orbits, although other orbital characteristics are also changed. However, from simple energy considerations it is readily seen that ΔV_5 and ΔV_6 do not produce the maximum depth of the belt nor the extremes in orbital period.

From the lower curve of Figure 15 it is possible to determine the maximum width of the belt.* An example will readily explain the use of this curve.

Given (from previous examples)

$$\frac{\Delta V}{V} = 0.00132$$

$$R_o = 26.925 \times 10^6 \text{ ft}$$

From Figure 15

$$\frac{2W}{R_o} = .0026$$

$$2W = .0026\ (26.925 \times 10^6)$$

$$= 70,000 \text{ ft}$$

$$= 13.2 \text{ miles}$$

*See Appendix B for derivation of the equation.

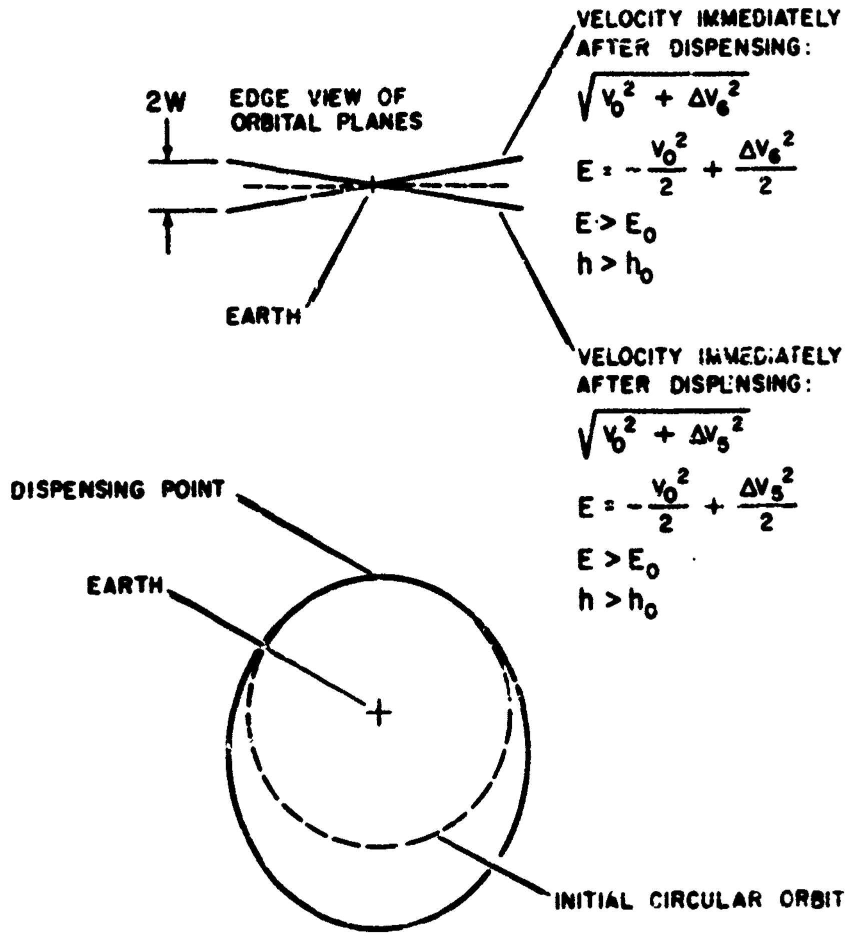


FIGURE 17

QUALITATIVE ORBITS AFTER PERTURBATION FOR ΔV_5 AND ΔV_6

5. Other Considerations

It is interesting to consider the result obtained when the chaff is dispensed from the orbital vehicle without a velocity component parallel to the original velocity vector. (All ΔV's are equal and perpendicular to the original velocity vector.) After dispensing, all dipoles will have the same energy and hence the same period (Equation 5). Instead of forming a belt, the chaff cloud will grow and change as a function of time in some peculiar cyclic manner, repeating this cycle each orbit. All the chaff will theoretically arrive back at the dispensing point at the same time. Thus as ΔV_1, ΔV_2, and all other velocity components parallel to V_o approach zero, the number of orbits required to produce an earth belt approaches infinity, as indicated by Figure 14.

VI. CONCLUSIONS

(1) The vapor-pressure technique is very effective for separating dipoles in a space-like environment.

(2) When dispensing fluid-saturated chaff in a low-pressure environment, the dipole velocity is approximately linear with the square root of the fluid vapor pressure.

(3) Chaff dispensed omnidirectionally from a vehicle in a circular geocentric orbit will form a belt around the earth.

(4) Chaff given a uniform dispensing velocity perpendicular to the original circular orbital velocity will produce a chaff cloud which grows and changes in some cyclic manner. The cycle will repeat once each orbit and the cloud theoretically will not form an earth belt.

APPENDIX A

TABULATION OF EXPERIMENTAL DATA AND RESULTS

Run No.	Fluid	Chaff	Vapor Pressure at Test Temp. lb/ft^2	$\frac{A}{m}$ $\frac{ft^2}{slug}$	$\sqrt{\frac{PA}{m}}$	Measured Dipole Velocity ft/sec	Remarks
A	0.65 cs Dow Corning	Aluminum 3/4"x0.036"	97.5	5000	698	37	
B	0.65 cs Dow Corning	Aluminum 3/4"x0.016"	103.0	2260	485	23	
C	0.65 cs Dow Corning	Aluminum 3/4"x0.016"	86.4	4850	647	-	Movies of test were blurred
D	Acetic Acid	Aluminum 3/4"x0.036"	39.1	5000	442	36	
E	0.65 cs Dow Corning	Aluminum 3/4"x0.016"	94.7	4850	678	34	
F	0.65 cs Dow Corning	Aluminum 4.5"x0.25"	97.5	4860	609	-	No velocity measurement was made
G	Acetic Acid	Aluminum 3/4"x0.036"	45.1	5000	475	14	
H	1.0 cs Dow Corning	Aluminum 3/4"x0.036"	7.5	5000	194	10	
I	Methyl Alcohol	Aluminum 3/4"x0.036"	278.5	5000	1179	47	
J	0.65 cs Dow Corning	Aluminum 4.5"x0.25"	95.3	4860	674	34	
K	Methyl Alcohol	Aluminum 3/4"x0.036"	334.0	5000	1292	47	
L	Methyl Alcohol	Aluminum 3/4"x0.036"	306.2	5000	1237	32	
M	Methyl Alcohol	Aluminum 3/4"x0.036"	342.3	5000	1308	38	Bursting difficulties encountered
N	Methyl Alcohol	Aluminum 4.5"x0.25"	292.3	4860	1192	-	Film of test ruined in developing
O	0.65 cs Dow Corning	Aluminum 4.5"x0.25"	96.7	4860	678	31	
P	Methyl Alcohol	Aluminum 3/4"x0.036"	342.3	5000	1309	28	[illegible] insure fluid
Q	Methyl Alcohol	Aluminum 3/4"x0.036"	361.8	5000	1345	-	Ejection was not recorded
R	Methyl Alcohol	Aluminum 3/4"x0.036"	320.0	5000	1265	43	
S	Methyl Alcohol	Aluminum 4.5"x0.25"	361.8	4860	1326	34	
T	Methyl Alcohol	Aluminum 3/4"x0.036"	361.8	5000	1345	48	
U	Carbon Tetrachloride	Aluminum 3/4"x0.036"	311.8	5000	1248	48	
V	Acetone	Aluminum 3/4"x0.036"	-	-	-	-	O-rings did not seal
W	None	Aluminum 3/4"x0.036"	-	5000	-	-	No velocity calculation made
X	Acetone	Aluminum 3/4"x0.036"	495	5000	1362	80	
Y	Acetone	Aluminum 3/4"x0.036	665	5000	1824	79	
AA	Acetone	Aluminum 3/4"x0.036"	637	5000	1785	77	
BB	1.0 cs Dow Corning	Aluminum 3/4"x0.036"	7.5	5000	194	9	
CC	0.65 cs Dow Corning	Glass 3/4"x[illegible]" dia.	80.6	1400	999	18	

APPENDIX B

Definition of Terms and Nomenclature used in Text and Appendix A

NOTE: Subscripts 1 through 6 refer to ΔV's 1 through 6 as defined by Figure 11 of text.

- μ - Geocentric orbital constant = $g_o R^2 = 140.99187 \times 10^{14}\ ft^3/sec^2$
- τ - Orbital period
- E - Total specific energy of orbit
- V_o - Dipole or vehicle velocity in the original circular orbit
- V - Dipole velocity after ejection
- R - Radius of earth
- R_o - Radius of original circular orbit measured from the earth's center
- ΔV - Velocity perturbation relative to the dispensing vehicle
- n - Number of orbits necessary to form a chaff belt around the earth
- a - Semi-major axis of elliptical orbit
- r - Radius of a point in elliptical orbit
- θ - Angular orientation of a point in orbit measured counterclockwise from the apse line of the orbit
- ϵ - Eccentricity of orbit
- h - Specific angular momentum
- b - Semi-minor axis of elliptical orbit
- W - Maximum cloud width
- φ - Angle between the orbits of dipoles dispensed with ΔV_5 and ΔV_6 and the original orbit

I. Derivation of n Expression

$$(1) \quad \tau_1 = \frac{2\pi\mu}{(-2E_1)^{3/2}}$$

and

$$(2) \quad \tau_2 = \frac{2\pi\mu}{(-2E_2)^{3/2}}$$

where:

$$(3) \quad E_1 = 1/2\, V_1^{\,2} - \frac{\mu}{R_o}$$

and

$$(4) \quad V_1 = V_o - \Delta V_1$$

$$(5) \quad E_2 = 1/2\, V_2^{\,2} - \frac{\mu}{R_o}$$

$$(6) \quad V_2 = V_o + \Delta V_2$$

But:

$$(7) \quad V_o^{\,2} = \frac{\mu}{R_o}$$

Applying (4), (6), and (7) to (3) and (5) and noting that $\left|\Delta V_1\right| = \left|\Delta V_2\right| = \Delta V$ we obtain:

$$(3') \quad E_1 = \frac{-V_o^{\,2} - 2\, V_o\, \Delta V + \Delta V^2}{2}$$

$$(5') \quad E_2 = \frac{-V_o^{\,2} + 2\, V_o\, \Delta V + \Delta V^2}{2}$$

Note: from (3) and (5) that $\left| E_2 \right| < \left| E_1 \right|$

Therefore:

$$\tau_2 > \tau_1$$

At the instant the earth chaff belt is complete

$$(8) \quad n_2\tau_2 = (n_2+1)\tau_1$$

Solving (8) for n_2

$$(9) \quad n_2 = \frac{\tau_1}{\tau_2-\tau_1}$$

Then substituting (1) and (2) into (9) and simplifying we obtain:

$$(9') \quad n_2 = \frac{1}{\left(\frac{E_1}{E_2}\right)^{3/2} - 1}$$

II. Proof of Generality of the "n" vs $\frac{\Delta V}{V_o}$ plot

Consider two sets of dipoles denoted by subscripts a and b. Assume they are ejected into orbit such that:

(1) $V_{oa} = k\, V_{ob}$

and $\Delta V_a = k\, \Delta V_b$

or $\frac{\Delta V_a}{V_{oa}} = \frac{\Delta V_b}{V_{ob}}$

Then from (9') of I in the appendix:

$$n_a = \frac{1}{\left(\frac{E_{1a}}{E_{2a}}\right)^{3/2} - 1}$$

(2)

$$n_b = \frac{1}{\left(\frac{E_{1b}}{E_{2b}}\right)^{3/2} - 1}$$

The number of orbits required to belt the earth is determined by ΔV_1 and ΔV_2. From equations (3') and (5') of I in the appendix:

$$E_{1a} = \frac{-V_{oa}^{\;2} - 2\, V_{oa}\, \Delta V_a + \Delta V_a^{\;2}}{2}$$

(3)

$$E_{2a} = \frac{-V_{oa}^{\;2} + 2\, V_{oa}\, \Delta V_a + \Delta V_a^{\;2}}{2}$$

$$E_{1b} = \frac{-V_{ob}^{\;2} - 2\, V_{ob}\, \Delta V_b + \Delta V_b^{\;2}}{2}$$

(4)

$$E_{2b} = \frac{-V_{ob}^{\;2} + 2\, V_{ob}\, \Delta V_b + \Delta V_b^{\;2}}{2}$$

Substitute (1) into (4):

$$(4') \quad \begin{aligned} E_{1b} &= k^2 \frac{\left[-V_{oa}{}^2 - 2\, V_{oa}\, \Delta V_a + \Delta V_a{}^2\right]}{2} = k^2\, E_{1a} \\ E_{2b} &= k^2 \frac{\left[-V_{oa}{}^2 + 2\, V_{oa}\, \Delta V_a + \Delta V_a{}^2\right]}{2} = k^2\, E_{2a} \end{aligned}$$

Substitute (4') into (2):

$$(2') \quad n_b = \frac{1}{\left(\dfrac{k^2 E_{1a}}{k^2 E_{2a}}\right)^{3/2} - 1} = \frac{1}{\left(\dfrac{E_{1a}}{E_{2a}}\right)^{3/2} - 1}$$

or $$n_b = n_a \text{ for } \frac{\Delta V_a}{V_{oa}} = \frac{\Delta V_b}{V_{ob}}$$

III. Derivation of Equation for Dimensionless Maximum Cloud Depth Parameter

From basic central force orbit equations:

$$(1) \quad a_2 = \frac{-\mu}{2E_2}$$

$$(2) \quad a_1 = \frac{-\mu}{2E_1}$$

Then the maximum depth of separation between the two orbits is given by:

$$2(a_2 - a_1) = -\mu \left(\frac{1}{E_2} - \frac{1}{E_1} \right)$$

or

$$2(a_2 - a_1) = 2\mu \left(\frac{1}{V_o^2 - 2V_o \Delta V - \Delta V^2} - \frac{1}{V_o^2 + 2V_o \Delta V - \Delta V^2} \right)$$

$$(3) \quad 2(a_2 - a_1) = \frac{8\mu\, V_o\, \Delta V}{\left[V_o^2 - 2V_o \Delta V - \Delta V^2 \right] \left[V_o^2 + 2V_o \Delta V - \Delta V^2 \right]}$$

Now note that:

$$R_o = \frac{\mu}{V_o^2}$$

Then the dimensionless maximum cloud depth parameter is defined as:

$$2(a_2 - a_1)/R_o$$

and is given by:

$$(4) \quad \frac{2(a_2 - a_1)}{R_o} = \frac{8}{\dfrac{V_o}{\Delta V} - \dfrac{6\Delta V}{V_o} + \left(\dfrac{\Delta V}{V_o} \right)^3}$$

IV. Proof of Apse Line Shift for Dipoles Dispensed with ΔV_3 and ΔV_4

The parametric equation for an elliptical conic section is given by:

(1) $\frac{r}{a} = \frac{1 - \epsilon^2}{1 + \epsilon \cos \theta}$

or solving for $\cos \theta$:

(1') $\cos \theta = (1 - \epsilon^2) \frac{a}{r\epsilon} - \frac{1}{\epsilon}$

But ϵ is given by:

(2) $\epsilon = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$

and: (3) $a = -\frac{\mu}{2E}$

Substitute (2) and (3) into (1').

(1") $\cos \theta = \frac{\frac{h^2}{\mu} - r}{r\sqrt{1 + \frac{2Eh^2}{\mu^2}}}$

Then at the instant of velocity perturbation:

(4) $r = R_o = \frac{\mu}{V_o^2}$

and for dipoles dispensed with ΔV_3 and ΔV_4:

(5) $\left|\Delta V_3\right| = \left|\Delta V_4\right| = \Delta V$

(6) $V_3^2 = V_4^2 = V_o^2 + \Delta V^2$

(7) $E_3 = 1/2\ V_3^2 - \frac{\mu}{R_o}$

(8) $E_4 = 1/2\ V_4^2 - \frac{\mu}{R_o}$

Applying (4), (5), (6) to (7) and (8) we find:

(9) $E_3 = E_4 = \frac{-V_o^2 + \Delta V^2}{2}$

Since the angular momentum is unchanged by the velocity perturbation

(10) $h = V_o\ R_o = \sqrt{\mu R_o}$

Substitute (4), (9), and (10) into (1"):

$$(1''')\ \cos\theta = \frac{\frac{\mu R_o}{\mu} - R_o}{R_o\sqrt{1 + 2\,\frac{(-V_o^2 + \Delta V^2)\ V_o^2\ R_o^2}{2\,\mu^2}}}$$

or $$\cos\theta = \frac{0}{\frac{\Delta V}{V_o}} = 0$$

or $$\theta = \pm 90°$$

V. Derivation of Equation for Dimensionless Maximum Cloud Width Parameter

Maximum cloud width may be expressed by:

(1) $2W = 2b \sin \varphi$

where

(2) $\varphi = \mathrm{Tan}^{-1} \dfrac{\Delta V}{V_o}$

where $\Delta V = \left| \Delta V_5 \right| = \left| \Delta V_6 \right|$

(3) $b = a\sqrt{1 - \epsilon^2}$

(4) $\epsilon = \sqrt{1 + 2 \dfrac{Eh^2}{\mu^2}}$

(5) $h = \sqrt{V_o^2 + \Delta V^2}\, R_o$

(6) $E = 1/2 \left(V_o^2 + \Delta V^2 \right) - \dfrac{\mu}{R_o}$

or $E = \dfrac{-V_o^2 + \Delta V^2}{2}$

(7) $a = \dfrac{-\mu}{2E} = \dfrac{\mu}{V_o^2 - \Delta V^2}$

Substitute (5) and (6) into (4).

(4') $\epsilon = \left(\dfrac{\Delta V}{V_o} \right)^2$

Substitute (7) and (4') into (3).

$$(3')\quad b = \frac{\mu}{V_o^2}\sqrt{\frac{V_o^2 + \Delta V^2}{V_o^2 - \Delta V^2}}$$

Also note that:

$$(8)\quad \sin\varphi = \frac{\Delta V}{\sqrt{V_o^2 + \Delta V^2}}$$

Substitute (3') and (8) into (1).

$$(1')\quad 2W = \frac{2\mu\Delta V}{V_o^2\sqrt{V_o^2 - \Delta V^2}}$$

Define the dimensionless maximum cloud width parameter [illegible]:

$$\frac{2W}{R_o}$$

Dividing (1') by R_o, we obtain:

$$(9)\quad \frac{2W}{R_o} = \frac{2\Delta V}{\sqrt{V_o^2 - \Delta V^2}}$$